MathBootCamps Problem Pack

Calculus: The Power Rule for Integrals

*****PREVIEW*****

First edition – published 2017.

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For permissions and other requests:

contact@mathbootcamps.com

Contents

Introduction

This problem pack is designed so that you can practice problems! That means that it doesn't go into too much theory (just enough to remind you of the rules) and instead focuses on the mechanics of how to answer the math questions. You will however, still find useful tips and examples before each set of problems.

How to use this problem pack

To get the most of this problem pack, you should work through each and every problem, checking answers as you go. A good idea would be to try three or four problems, then check to see how you are doing. This will help you avoid repeating the same mistake too many times.

Also be sure to review the tips before each section, since that can help you make sure your first approach to a question is correct!

The basics

For $n \neq -1$:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

In words: "take the exponent, add 1 to it, and then divide by that number".

Example: $\int x^2 dx = \frac{x^3}{3} + C$

Tips

- Don't forget that you can take a constant out before applying the rule. For example: $\int 3x^4 dx = 3\left(\frac{x^5}{5}\right) + C = \frac{3x^5}{5} + C$
- You can also break integrals up over addition and subtraction. In other words, you can apply the rule to each piece of a function as long as the pieces are added or subtracted.

  For example: $\int 5x^4 - 2x^2 dx = 5\left(\frac{x^5}{5}\right) - 2\left(\frac{x^3}{3}\right) = x^5 - \frac{2x^3}{3} + C$
- If there is no exponent, it is assumed to be 1. For example:

  $\int x\, dx = \frac{x^2}{2} + C$ since $x = x^1$
- The integral of a constant is that constant times the variable. For example: $\int -3\, dx = -3x + C$ and $\int 5\, dt = 5t + C$
- Finally, this rule applies no matter what the exponent is (except –1). It could be a fraction, a decimal or a negative number! For example:

  $\int x^{0.1} = \frac{x^{1.1}}{1.1} + C$ and $\int x^{\frac{1}{4}}\, dx = \frac{x^{\frac{1}{4}+1}}{\frac{1}{4}+1} + C = \frac{x^{\frac{1}{4}+\frac{4}{4}}}{\frac{1}{4}+\frac{4}{4}} + C = \frac{x^{\frac{5}{4}}}{\frac{5}{4}} + C = \frac{4}{5}x^{\frac{5}{4}} + C$

Practice 1

Find the indefinite integral for each of the following functions.

1. $10x^4$

2. $-5x^2$

3. $17x$

4. $-\frac{1}{2}x+2$

5. $\frac{x^4}{10}-3x$

6. $-\frac{2}{5}x+3x^2$

7. $-3x^{\frac{1}{2}}$

8. $4x^{0.2}+3x^{0.5}$

9. $-x^{\frac{1}{2}}+3$

10. $\frac{2}{7}x^5-\frac{4}{7}x^{\frac{1}{7}}+9$

11. $12x^3-14x^5+1$

12. $\frac{15}{4}x^{\frac{1}{3}}-1$

13. $2x^2+x^4-\frac{3}{5}x^3$

14. $2x^{1.5}-1.8x^{2.4}$

15. $-x^4-x^{\frac{1}{4}}$

Dealing with fractions and negative exponents

Fractions with a single term in the denominator can be "broken up" over addition and subtraction.

The general rule is: $\frac{A+B}{C}=\frac{A}{C}+\frac{B}{C}$

Therefore: $\frac{3-x}{2}=\frac{3}{2}-\frac{x}{2}$ and $\frac{4x+1}{x^2}=\frac{4x}{x^2}+\frac{1}{x^2}=\frac{4}{x}+\frac{1}{x^2}$

When a variable in the denominator has an exponent, that can be rewritten using a negative exponent.

The rule for this is: $x^{-n}=\frac{1}{x^n}$

Therefore: $\frac{4}{x}+\frac{1}{x^2}=4x^{-1}+x^{-2}$ and $\frac{5}{x^2}+\frac{1}{x^3}=5x^{-2}+x^{-3}$

Once you have a function written this way, you can find the integral using the power rule.

Example

$$\int \frac{3x+1}{x^4}\,dx=\int \frac{3x}{x^4}+\frac{1}{x^4}\,dx=\int \frac{3}{x^3}+\frac{1}{x^4}\,dx=\int 3x^{-3}+x^{-4}\,dx$$

$$=3\left(\frac{x^{-3+1}}{-3+1}\right)+\frac{x^{-4+1}}{-4+1}+C$$

$$=3\left(\frac{x^{-2}}{-2}\right)+\frac{x^{-3}}{-3}+C$$

$$=-\frac{3}{2}x^{-2}-3x^{-3}+C$$

The answer can also be written using fractions: $-\frac{3}{2}x^{-2}-3x^{-3}+C=-\frac{3}{2x^2}-\frac{3}{x^3}+C$

Tips

- Only the part with a negative exponent is written in the denominator when rewriting your answer as a fraction.

  For example: $\frac{2}{5}x^{-2} = \frac{2}{5x^2}$

- The following are equivalent. You should check with your teacher to see which version he or she prefers.

  $\frac{1}{2}x^5 = \frac{x^5}{2}$

- Remember that a negative sign in the top or bottom of a fraction can be brought out front.

  $-\frac{2}{3} = \frac{-2}{3} = \frac{2}{-3}$

- Finally, and this is very important, while you can break a fraction up over a single term in the denominator, you cannot break up over multiple terms!

  $\frac{2}{x+5} \neq \frac{2}{x} + \frac{2}{5}$ and $\frac{3+x}{x^2-2} \neq \frac{3}{x^2} - \frac{x}{2}$

*****END PREVIEW******